

SPURIOUS PRODUCT ZONE CHART
A PRACTICAL APPROACH TO FREQUENCY
CONVERSION SYNTHESIS

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ABSTRACT

Mixer charts and computerized spurious product search are commonly utilized techniques for determining the spurious/intermodulation spectrum of communication systems or equipment utilizing frequency conversion. Both methods require intuition, a trial-and-error approach, and lack the adaptability to illustrate computer graphics. The spurious product zone charting method provides a clear, quick, and concise mathematical/graphical presentation of spurious/intermodulation (IM) products resulting from conversion processes. It adapts easily to computer aided design graphic displays or to small calculators with alphanumeric readouts.

INTRODUCTION

Spurious zone charting was discussed and utilized in previous papers^{3,4}. Practical examples of zone charting of specific spurious/IM products are treated by Spilker⁴ in connection with the frequency converter intermediate frequency (IF) selection problem at X-band (7.25-8.4 GHz) in a satellite communication system.

Spurious product zone charting was prompted by project related needs such as integrated ground terminal development for SHF satellite communication bands (20/30 GHz) and analysis and design of frequency hopping/spread spectrum conversion system architectures. The following plan was utilized: (1) A survey was carried out to classify all typical frequency conversion processes; (2) Further study resulted in the classification of spurious/IM products; (3) Theoretical baseline and associated rules were established and zone equations were derived; and (4) A suitable coordinate system was selected, and zonal relationships implemented using an HP9836 colorgraphic desktop computer.

DISCUSSION

Figure 1 is a summary classification of frequency conversion processes. A typical frequency conversion stage is shown indicating variable parameters. In downconversion F_{ic} is a known parameter and F_{oc} is selected; in upconversion the reverse applies. B_i , B_l and B_o (input signal, local oscillator and output bandwidth respectively) are also known parameters.

The analysis procedure consists of computing the zone limits (graphing them on a zone chart if available) and selecting the proper frequency output center (F_{oc}) or frequency input center (F_{ic}) for upconversion such that certain undesired spurious/IM products are avoided.

For multiple stages, one zone chart is used per stage. Thus the unknown F_{oc} of each preceding stage becomes the known F_{ic} of the following stage as center frequencies are progressively selected. The amount of work associated with multistage frequency conversion can be reduced by using complementary zone charts. A reverse mapping is used in the second stage, as spurious product zones are mapped in to the F_{ic} region and F_{oc} is treated as a known value. Thus the zones become common to both stages and selection of an IF accomplishes both objectives simultaneously, requiring only *one chart per two stages*.

Figure 2 is a typical zone chart for Case 2 D/C LSLI.

The known parameters are shown in the bottom field of the graph. Image zones are slightly offset vertically for easy distinction from the real zones. The first group of zonal lines emerging from the vertical axis

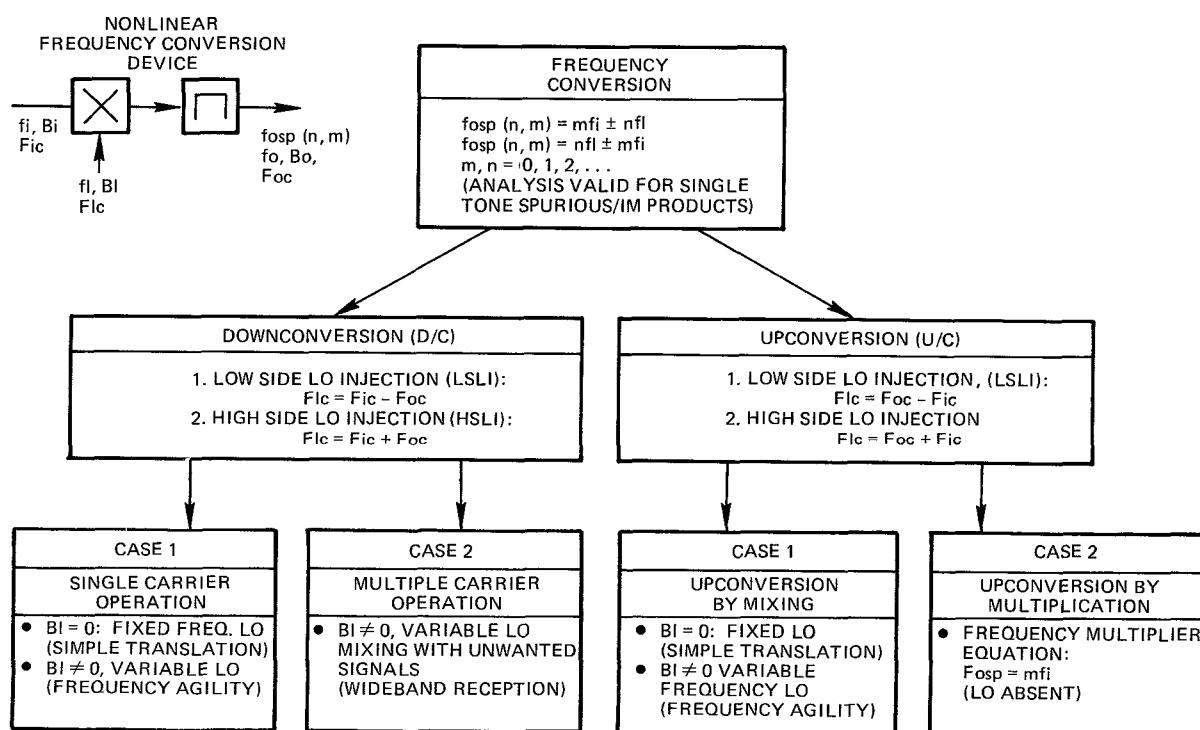


Figure 1. Classification of Frequency
Conversion Processes

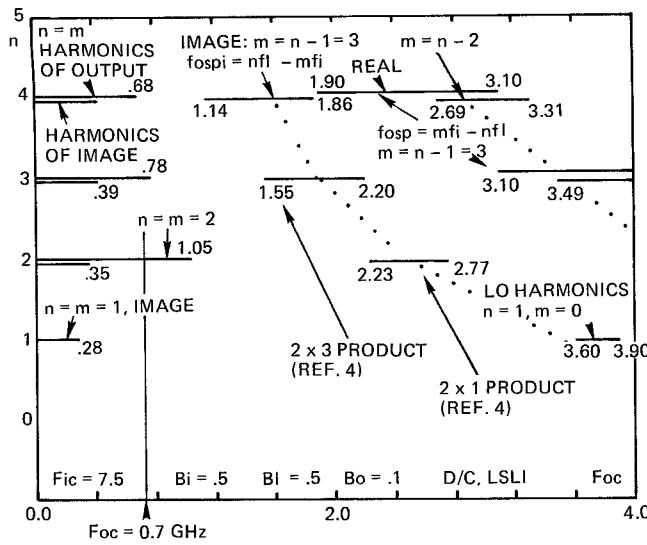


Figure 2. Spurious Zone Chart for Downconverter Low-Side LO Injection (D/C LSLI), Multiple Carrier Inputs

($Foc = 0$) correspond to $m = n$, the second $m = n-1$, etc. Using colorgraphics, each zone type can be plotted in a different color for better distinction. The order O is generally defined as $O = m + n$. The results agree with (4) with the only exception being an additional 2×2 product is also present here with an upper boundary of 1050 MHz. Consequently, if common IF is the objective to select for both upconversion (U/C) and downconversion (D/C) the allowable region of IF falls between 1050 and 1450 MHz instead of 600 and 1450 MHz. This product is the result of multiple carrier reception when undesired carriers are present in the input band between 7375 and 7425 MHz. LO in this case is tuned to 7050 MHz and the desired carrier is at $7050 + 700 = 7750$ MHz (upper band edge). The frequency plan and resulting input spectrum is shown in Figure 3. Spurious contamination due to this product is verified easily as follows:

Select $Foc = 0.700$ GHz as shown in Figure 2 by the vertical line which intersects the 2×2 product zone. The following apply:

$$\begin{aligned} Foc &= 0.7 \text{ GHz}, B = 0.1 \text{ GHz} \therefore fo = 0.650 \text{ to } 0.750 \text{ GHz} \\ Fic &= Fic - Foc = 6.8 \text{ GHz}, BI = 0.5 \text{ GHz} \therefore fi = 6.55 \text{ to } 7.05 \text{ GHz} \\ Fic &= 7.5 \text{ GHz}, Bi = 0.5 \text{ GHz} \therefore fi = 7.25 \text{ to } 7.75 \text{ GHz} \end{aligned}$$

The output spurious range is:

$$\begin{aligned} Fosp1 &= 2(7.375 - 7.050) = 0.650 \text{ GHz} \text{ (lower end of output band)} \\ Fosp2 &= 2(7.425 - 7.050) = 0.750 \text{ GHz} \text{ (upper end of output band)} \end{aligned}$$

Therefore, spurious 2×2 products contaminate the output band at this LO tuning position. Thus, verification is complete.

Space constraints prevented full discussion of this subject including zone charts for upconversion. Spurious zone charts for upconversion are similar in format, except that the dependent variable is m and the independent variable is Fic .

Based on this zone chart, one can conclude that, if the IF for downconversion is selected between 1050 and 1550 MHz, then the lowest order product is seventh ($n = 4, m = 3$) in the downconversion process.

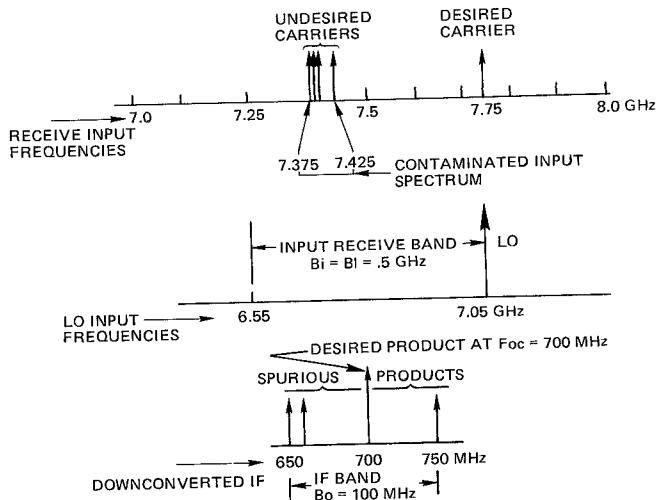


Figure 3. Frequency Plan and Contaminated Input Spectrum for the Case of Multiple Carrier D/C LSLI

APPENDIX

a. **Baseline.** Define the spurious/IM products occupying spurious zones at the output of a nonlinear frequency conversion device by the equations:

$$fosp(m,n) = nfi \pm mfi \quad (1a)$$

$$\text{or } fosp(m,n) = mfi \pm nfi, n=0,1,2, \dots, m=0,1,2, \dots \text{ (positive integers)} \quad (1b)$$

where fi and fl are instantaneous values of input and LO frequencies, both are varying quantities in general. All frequencies are positive and real. The products defined by (1a) and (1b) are considered as "real" and "images" relative to one another when negative signs are used.

The following limits are imposed upon the range of variation of fl and fi :

$$Flc - \frac{Bl}{2} < fl < Flc + \frac{Bl}{2}, Fic - \frac{Bi}{2} < fi < Fic + \frac{Bi}{2} \quad (2)$$

where Flc and Fic are input and LO center frequencies and Bl and Bi are input and LO bandwidths. As fl and fi vary within these limits, $fosp$ varies over a wider range, determined by the harmonic numbers n and m . We are interested in those products that pass through or dwell within the output bandwidth Bo . Let the range of spurious/IM contaminated output center frequencies, (Foc , or for upconversion, Fic), be bounded by $Foc(\min)$ at the low end and $Foc(\max)$ at the high end of the spurious zone. These boundaries are represented functionally as follows:

$$\begin{aligned} Foc(\min, \max) &= g[Fic, Bi, Bl, Bo, m, n] \text{ for downconversion} \\ Fic(\min, \max) &= g[Foc, Bo, Bl, Bi, m, n] \text{ for upconversion} \end{aligned}$$

Inequality (3) below applies with appropriate signs for each case in deriving all zonal boundary points:

$$Foc - \frac{Bo}{2} < n(Flc \pm \frac{Bl}{2}) \pm m(Fic \pm \frac{Bi}{2}) < Foc + \frac{Bo}{2} \quad (3)$$

Suppose, Foc is selected such that $Foc(\min) < Foc(n, m)$

$< Foc(\max)$, then Foc is a solution of inequality (3) and is within the spurious zone.

b. **Normal Zone.** The zonal solution due to the real crossproducts of input for Case 3, D/C LSLI is given by (4):

$$Foc(\min, \max) = \frac{(n-m)Fic \pm 0.5[(n-m)Bl + m(Bl - Bi) + Bo]}{n-1} \quad m < n, n=2,3, \dots, m=1,2, \dots \quad (4)$$

The special case: $n = m$ corresponds to the harmonics of output, given by (5):

$$Foc(\min, \max) = 0, 0.5 \frac{m(Bi-BI) + Bo}{m-1} \quad m = 2, 3, \dots \quad (5)$$

It can also be proven that the image of each of the above real spurious products has an image zone with the limits $Foci(\min, \max)$ according to (6).

$$Foci(\min, \max) = \frac{n-1}{n+1} \cdot Foc(\min, \max), \quad (6)$$

Equations (4) through (6) are applicable to single carrier operation, D/C LSLI.

Zonal solutions were developed to include the following spurious/IM zones.

- a. $n=0, m=1, 2, 3, \dots$ (input leakage and its harmonics).
- b. $m=0, n=1, 2, 3, \dots$ (LO leakage and its harmonics).
- c. Image of the input signal and its crossproducts and harmonics.
- d. Unwanted sideband, its image, crossproducts and harmonics.

Spurious products in the multicarrier case are generated when undesired signals are present within the input band of a receiver classified as Case 2, D/C LSLI as shown on Figure 1. The zone equation for this case is:

$$Foc(\min, \max) = \frac{(n-m) Fic \pm 0.5 (Bo+nBI+mBi)}{n-1} \quad n > m, n=2, 3, 4, \dots m=1, 2, \dots \quad (7)$$

where the above equation represents the real spurious zones of cross-products. Case 2 typically applies to wideband multiple carrier signal reception. The spurious zones are the widest for this case, yielding the least available spurious free territory; comparing (7) with (4) should verify this. Considering U/C, LSLI, when the algebraic sign in (1) is positive, the zonal solution is:

$$Fic(\min, \max) = \frac{Foc(n-1) \pm 0.5 (Bo+nBI+mBi)}{n-m} \quad m < n, n=2, 3, \dots m=1, 2, \dots \quad (8)$$

Equation (8) gives the spurious zones of the crossproducts between input signal and LO. A special case of the above equation is $n = 1, m < n$ resulting in the following zonal boundaries:

$$Fic(\max) = \frac{0.5 (Bo+BI+mBi)}{m-1}, \quad m=2, 3, \dots \quad Fic(\min) = 0 \quad (9)$$

Equation (9) represents spurious zones due to mixing products between the LO and harmonics of the input signal ($fosp = fI + mfi$).

c. *Complementary Zones and Null Zones.* The concept of complementary zones is useful when two or more conversion stages should be analyzed. Consider the spurious zones of the second stage of a two-stage unit in a multiple downconversion scheme having the form: $Foc(\max, \min) = g[Fic, Bi, BI, Bo, n, m]$. The complementary mapping is defined by the inverse relation:

$$\bar{Fic}(\max, \min) = h[Foc, Bi, BI, Bo, n, m] \quad (10)$$

Both equations designate the same conversion process except the roles of variables Foc and Fic are interchanged. Based on the above, the complementary form of (4) is:

$$\bar{Fic}(\max, \min) = \frac{(n-1)Foc \pm 0.5[(n-m)Bi+m(Bi-BI)+Bo]}{n-m} \quad n > m, n=2, 3, \dots, m=1, 2, \dots \quad (11)$$

Certain zones cannot be mapped simultaneously in the normal and complementary planes; these are called "null zones." For example, (9) having the functional form: $Foc(\max, \min) = g[Bi, BI, Bo, n, m]$ contains spurious zones in the normal region, and null zones in the complementary region. The null zones may also need to be displayed or identified if they are (or are close to being) in-band.

Suppose the normal zone of a first downconversion stage has the form: $Foc1(\max, \min) = g(Fic1, Bi1, Bo1, BI1, m, n)$, then the complementary zone of the second stage is expressed as:

$$\bar{Fic}2(\max, \min) = h[Foc2, Bi2, Bo2, BI2, m, n] \quad (12)$$

When the two stages are cascaded, the following commonality exists:

$$Foc1 = \bar{Fic}2 = FI \text{ and } Bo1 = Bi2 = BI \quad (13)$$

Parameters FI and BI are recognized as the intermediate frequency and bandwidth common to both stages; thus, the amount of work is reduced, since the zones of normal and complementary region can be overlaid and analyzed simultaneously for both stages.

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